A UNIFIED APPROACH TO THE DESIGN OF ORBIT FEEDBACK WITH FAST AND SLOW CORRECTORS

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Abstract

A unified control design is proposed to simultaneously determine the inputs to both the fast and slow arrays of correctors. By determining the interaction of the spatial subspaces of each array of correctors, spatial modes which require both fast and slow correctors can be identified. For these modes, a mid-ranging control technique is proposed to systematically allocate control action for each corrector. The mid-ranging control technique exploits the different dynamic characteristics of the correctors to ensure that the two arrays of actuators work together and avoid saturation of the fast correctors. Simulation results for the Diamond Storage Ring are presented.

INTRODUCTION

In this paper, an approach to electron orbit controller design is presented where there are two arrays of actuators with different dynamics. The aim is to design a controller which uses both corrector arrays to meet the electron beam stability requirements so that: the time taken for computation of the control action is not greater than that of the single array system, the tuning for performance is intuitive and the specific dynamics and constraints of each array are addressed. In order to meet these goals, a method of exploiting the knowledge of the spatial responses of the arrays of correctors is used to determine the interaction between the controllable subspaces. As a result the problem can be decomposed into a series of single-input, single-output (SISO), two-input, single-output (TISO) and two-input, two-output (TITO) problems. As a consequence, the online computation is minimised by selecting appropriate control directions when subspaces of the two arrays overlap. In particular, if the control directions of the two arrays are orthogonal, then the problem can be interpreted as a decoupled structure (i.e. two SISO structures) and SISO design techniques can be applied. If however, the control directions align, the problem has a TISO structure, and mid-ranging control is proposed. The term mid-ranging control typically refers to the class of control problems where two actuators are manipulated to control one measured variable. Furthermore there is the condition that one input should return to its midpoint or some setpoint. The inputs usually differ in their dynamic effect on the output and in the relative cost of manipulating each one, with the fast input normally being more costly to use than the slow input [1]. Therefore mid-ranging control schemes seek to manipulate both inputs upon an upset but then gradually reset or mid-range the fast input to its desired setpoint. Mid-ranging control therefore is suitable for the fast and strong corrector problem. Though it may be possible to manipulate each actuator separately, for electron beam control it is desirable to simultaneously manipulate both inputs, as the strong correctors are bandwidth limited. Additionally, an important characteristic of mid-ranging applications is that input constraints on the faster input are avoided. There are several approaches to designing mid-ranging controllers [1], but in [2] an IMC structure is used, which is adopted in this paper since it is consistent with the design for a single array of actuators at Diamond Light Source [3].

MULTI-ARRAY CONTROLLER DESIGN

Subspace Interaction

For a storage ring electron orbit control system with two arrays of actuators where N_f and N_s are the number of actuators in each array, the dynamics of each array $g_{s,f}(z)$ are given by a first order plus delay transfer function determined by the open loop bandwidth $a_{s,f}$ and delay $\tau_{s,f}$ [4], and without loss of generality, it is assumed that:

1. $g_s(1) = 1$ and $g_f(1) = 1$ i.e. the DC gains of the dynamics can be taken as unity.
2. the dynamics of $g_f(z)$ are faster than $g_s(z)$.
3. $N_s \leq N_f$.
4. all the actuators of a given array have the same dynamics.

The position measured at M BPMs is described by

$$Y(z) = g_s(z)R_sU_s(z) + g_f(z)R_fU_f(z) + D(z) \quad (1)$$

where $U_s(z)$ and $U_f(z)$ represent the inputs applied to the two distinct arrays of actuators and $D(z)$ represent the disturbances acting on the electron beam. The response matrices for each array is represented by R_s and R_f where $N_s = \text{rank}(R_s)$ and $N_f = \text{rank}(R_f)$. The response matrices can each be expressed in terms of reduced singular value decompositions so that,

$$R_s = \Phi_s\Sigma_s\Psi_s^{\mathrm{T}}, \qquad R_f = \Phi_f\Sigma_f\Psi_f^{\mathrm{T}} \quad (2)$$

where $\Phi_s \in \mathbb{R}^{M\times N_s}$, $\Sigma_s \in \mathbb{R}^{N_s\times N_s}$, $\Psi_s \in \mathbb{R}^{N_s\times N_s}$, $\Phi_f \in \mathbb{R}^{M\times N_f}$, $\Sigma_f \in \mathbb{R}^{N_f\times N_f}$ and $\Psi_f \in \mathbb{R}^{N_f\times N_f}$. The columns of Ψ_s and Ψ_f in Eq. 2 represent the controllable subspaces of the response matrices and even though R_s and R_f are independent, there may be some overlap between the controllable subspaces and the relationship between the subspaces of R_s and R_f can be found by comparing the subspaces of Φ_s and Φ_f which are both orthogonal and can be determined from Algorithm 12.4.3 in [5] such that

$$\Phi_f^{\mathrm{T}}\Phi_s = A \quad (3)$$

and the singular value decomposition of A is given by

$$A = \Phi_A\Sigma_A\Psi_A^{\mathrm{T}} \quad (4)$$

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with left matrix, $\Phi_A \in \mathbb{R}^{N_f \times N_s}$, right matrix $\Psi_A \in \mathbb{R}^{N_s \times N_s}$ and $\Sigma_A \in \mathbb{R}^{N_s \times N_s}$ such that

$$\Sigma_A = \mathrm{diag}\{\cos(\theta_{A_1}), \ldots, \cos(\theta_{A_{N_s}})\} \quad (5)$$

where θ_{A_i} is the angle between the principal vectors spanning Φ_f and Φ_s which span the controllable subspaces of R_s and R_f.

Internal Model Control Multi-array Design

An IMC structure for such a multi-array system results in a closed loop response given by

$$\begin{aligned} Y(z^{-1}) = \Big(I - g_s(z^{-1}) R_s \tilde{Q}_s(z^{-1}) \\ - g_f(z^{-1}) R_f \tilde{Q}_f(z^{-1})\Big) D(z) \end{aligned} \quad (6)$$

(for a zero setpoint) which can be expressed as,

$$Y(z) = \left(I - [\Phi_s \quad \Phi_f] \begin{bmatrix} g_s(z)\Sigma_s \Psi_s^{\mathrm{T}} \tilde{Q}_s(z) \\ g_f(z)\Sigma_f \Psi_f^{\mathrm{T}} \tilde{Q}_f(z) \end{bmatrix}\right) D(z) \quad (7)$$

where

$$\begin{aligned} \tilde{Q}_s(z) &= \Psi_s \Sigma_s^{-1} Q_s(z) \tilde{\Phi}_s \\ \tilde{Q}_f(z) &= \Psi_f \Sigma_f^{-1} Q_f(z) \tilde{\Phi}_f \end{aligned} \quad (8)$$

for some diagonal $Q_s(z)$ and $Q_f(z)$ and for $\tilde{\Phi}_s \in \mathbb{R}^{N_s \times M}$ and $\tilde{\Phi}_f \in \mathbb{R}^{N_f \times M}$. Therefore Eq. 7 can be expressed as

$$Y(z) = \left(I - [\Phi_s \quad \Phi_f] \begin{bmatrix} g_s(z) Q_s(z) \tilde{\Phi}_s \\ g_f(z) Q_f(z) \tilde{\Phi}_f \end{bmatrix}\right) D(z^{-1}) \quad (9)$$

and for steady state

$$Y_{ss} = \left(I - [\Phi_s \quad \Phi_f] \begin{bmatrix} \tilde{\Phi}_s \\ \tilde{\Phi}_f \end{bmatrix}\right) D_{ss} \quad (10)$$

since the diagonal transfer function matrices $Q_f(z)$ and $Q_s(z)$ are chosen to have unity DC gain. Pre-multiplying both sides by $[\Phi_s \quad \Phi_f]^{\mathrm{T}}$, projects the response into modal space so that

$$\begin{bmatrix} \Phi_s^{\mathrm{T}} \\ \Phi_f^{\mathrm{T}} \end{bmatrix} Y_{ss} = \begin{bmatrix} \Phi_s^{\mathrm{T}} \\ \Phi_f^{\mathrm{T}} \end{bmatrix} D_{ss} - \begin{bmatrix} \Phi_s^{\mathrm{T}} \\ \Phi_f^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \Phi_s & \Phi_f \end{bmatrix} \begin{bmatrix} \tilde{\Phi}_s \\ \tilde{\Phi}_f \end{bmatrix} D_{ss} \quad (11)$$

and

$$\begin{bmatrix} \tilde{\Phi}_s \\ \tilde{\Phi}_f \end{bmatrix} = K \begin{bmatrix} \Phi_s^{\mathrm{T}} \\ \Phi_f^{\mathrm{T}} \end{bmatrix} \quad (12)$$

is defined such that K is a decoupling matrix, with $K_{11} \in \mathbb{R}^{N_s \times N_s}$, $K_{12} \in \mathbb{R}^{N_s \times N_f}$, $K_{21} \in \mathbb{R}^{N_f \times N_s}$ and $K_{22} \in \mathbb{R}^{N_f \times N_f}$. From Eq. 11 and Eq. 12,

$$\bar{Y}_{ss} = \begin{bmatrix} \Phi_s^{\mathrm{T}} \\ \Phi_f^{\mathrm{T}} \end{bmatrix} Y_{ss}, \qquad \bar{D}_{ss} = \begin{bmatrix} \Phi_s^{\mathrm{T}} \\ \Phi_f^{\mathrm{T}} \end{bmatrix} D_{ss} \quad (13)$$

and

$$\bar{Y}_{ss} = \left(I - \begin{bmatrix} I & \Phi_s^{\mathrm{T}} \Phi_f \\ \Phi_f^T \Phi_s & I \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\right) \bar{H}_{ss}. \quad (14)$$

From the relationship in Eq. 3, then

$$\begin{bmatrix} I & \Phi_s^T \Phi_f \\ \Phi_f^T \Phi_s & I \end{bmatrix} = \begin{bmatrix} I & A \\ A^{\mathrm{T}} & I \end{bmatrix} \quad (15)$$

and given Eq. 4

$$\begin{aligned} &\begin{bmatrix} I & \Phi_s^T \Phi_f \\ \Phi_f^T \Phi_s & I \end{bmatrix} = \\ &\begin{bmatrix} \Phi_A & \\ & \Psi_A \end{bmatrix} \left[\begin{array}{cc|c} I & 0 & \Sigma_A \\ 0 & I & 0 \\ \hline \Sigma_A & 0 & I \end{array}\right] \begin{bmatrix} \Phi_A & \\ & \Psi_A \end{bmatrix}^{\mathrm{T}}. \end{aligned} \quad (16)$$

The ideal design would be to make K equal to the inverse of the matrix in Eq. 16 and because it has a block diagonal structure, this gives

$$K = \begin{bmatrix} \Phi_A & \\ & \Psi_A \end{bmatrix} \tilde{K} \begin{bmatrix} \Phi_A & \\ & \Psi_A \end{bmatrix}^{\mathrm{T}} \quad (17)$$

where

$$\tilde{K} = \begin{bmatrix} \tilde{\Sigma}_A & 0 & \tilde{\Sigma}_A \\ 0 & I & 0 \\ \tilde{\Sigma}_A & 0 & \tilde{\Sigma}_A \end{bmatrix} \odot \begin{bmatrix} I & 0 & -\Sigma_A \\ 0 & I & 0 \\ -\Sigma_A & 0 & I \end{bmatrix} \quad (18)$$

and $\tilde{\Sigma}_A \in \mathbb{R}^{N_s \times N_s}$ is a diagonal matrix with elements

$$\left[\tilde{\Sigma}_A\right]_{ii} = \frac{1}{1 - \cos^2 \theta_{A_i}} \quad (19)$$

and $\odot$ denotes element-wise multiplication. Difficulties with this choice arise when $\cos(\theta_{A_i}) \approx 1$ which occurs when the angle between the subspaces is small. Instead, heuristic choices for the elements of $\tilde{K}$ can be made, which are described below:

1. Define the following:

$$\begin{aligned} \tilde{K}_{11} &= \begin{bmatrix} \tilde{\Sigma}_A & 0 \\ 0 & I \end{bmatrix}, \qquad \tilde{K}_{12} = \begin{bmatrix} -\tilde{\Sigma}_A \Sigma_A \\ 0 \end{bmatrix} \\ \tilde{K}_{21} &= \begin{bmatrix} -\tilde{\Sigma}_A \Sigma_A & 0 \end{bmatrix}, \qquad \tilde{K}_{22} = \tilde{\Sigma}_A \end{aligned} \quad (20)$$

2. When $\cos(\theta_{A_i}) = 0$, the directions $\Phi_s(:, i)$ and $\Phi_f(:, i)$ are orthogonal so that the system is considered as a decoupled two-input, two-output system i.e. two SISO structures. The relationships in Eq. 20 give this automatically where the corresponding elements of $\tilde{K}_{11}$ and $\tilde{K}_{22}$ are set to 1 and $\tilde{K}_{12}$ and $\tilde{K}_{21}$ are set to 0 so that the two directions are controlled independently.
3. When $0 < \cos(\theta_{A_i}) < 1$, Eq. 20 automatically splits the effort between the two actuators.
4. When $\cos(\theta_{A_i})$ is close to 1, the directions $\Phi_s(:, i)$ and $\Phi_f(:, i)$ almost line up and choosing to control in one direction is appropriate. This is achieved by setting the corresponding elements of either $\tilde{K}_{11}$ or $\tilde{K}_{22}$ to 1 and the other to 0. In this case, the corresponding elements of $\tilde{K}_{12}$ and $\tilde{K}_{21}$ are set to 0.
5. When $\cos(\theta_{A_i}) = 1$, the directions $\Phi_s(:, i)$ and $\Phi_f(:, i)$ align, giving a TISO structure. In this case, mid-ranging control is appropriate due to the actuator characteristics and the corresponding diagonal elements of $\tilde{K}_{11}$ and $\tilde{K}_{22}$ are transfer functions designed using a mid-ranging control technique.

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Table 1: Angles Between Controllable Subspaces of R_s and R_f

	$\cos(\theta_{A_i})$	
$i = 1$	0.9978	0.9982
$i = 2$	0.9965	0.9896
$i = 3$	0.9503	0.8814
$i = 4$	0.8533	0.4833
$i = 5$	0.4750	0
$i = 6$	0.2290	0
$i = 7$	0	0

SIMULATION STUDY

In this section a simulation study is presented for the storage ring, where two arrays of actuators with different dynamics are considered. For this study, one array is considered to contain 'slow' corrector magnets which are bandwidth limited to 10 Hz ($a_s = 2\pi \times 10$) and the second array uses 'fast' corrector magnets which have a larger bandwidth of 700 Hz ($a_f = 2\pi \times 700$) but are amplitude limited. In each case the delay is taken as $\tau_{s,f} = 700$ μs. In this case, number of slow correctors are $N_s = 7$ and the number of fast correctors are $N_f = 165$. The frequency responses for the two arrays are shown in Fig. 1.

Choice of Control Directions

The cosine of the angles between the first 7 columns of Φ_s and Φ_f are determined by Eq. 4 and are listed in Table 1. The following strategy is used for control:

- When $\cos(\theta_{A_i}) \approx 1$, the control directions almost line up, so mid-ranging control is used. From Table 1, this is the case for $i = \{1, 2, 3, 4\}$ horizontally and $i = \{1, 2, 3\}$ vertically.
- When $\cos(\theta_{A_i})$ is small, the control effort is split using $\tilde{K}$ in Eq. 20. From Table 1, this is the case for $i = \{5, 6\}$ horizontally and $i = 4$ vertically.
- When $\cos(\theta_{A_i}) = 0$, the directions are decoupled and SISO structures are used. For $i = 7$ horizontally and $i = \{5, 6, 7\}$ vertically both the fast and slow actuators are used.

Mid-ranging Controller Design

For the TISO problem, the output is expressed as

$$Y(z) = \left(1 - g_s(z)q_{mr_s}(z) - g_f(z)q_{mr_f}(z)\right) D(z)+ \left(g_s(z)q_{p_s}(z)) + g_f(z)q_{p_f}(z)\right) U_r(z) \tag{21}$$

where $g_s(z)$ and $g_f(z)$ are the slow and fast process models respectively and $q_{mr_s}(z)$ and $q_{mr_f}(z)$ are the associated IMC controllers. The mid-ranging objective is to use both inputs to control Y and return the fast input to its setpoint U_r. Pre-filters $q_{p_s}(z)$ and $q_{p_f}(z)$ are included to obtain a desired response from U_r to Y [2]. Therefore, the corresponding elements of $\tilde{K}_{11}$ and $\tilde{K}_{22}$ in Eq. 20 are $q_{mr_s}(z)/q_{s_i}(z)$ and $q_{mr_f}(z)/q_{f_i}(z)$ respectively. The control signals to the two actuators are

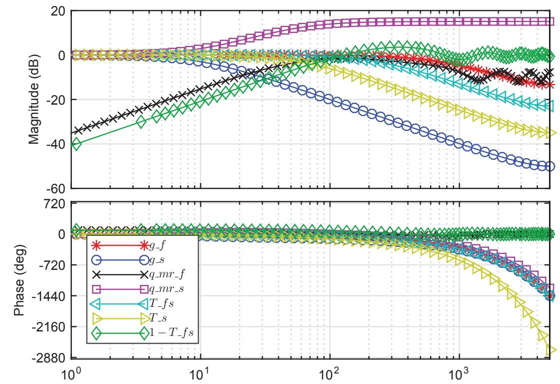

Figure 1: Frequency responses of fast ('∗' red) and slow actuators ('o' blue), fast ('x' black) and slow ('□' magenta) IMC controllers, fast and slow together complementary sensitivity ('◁' cyan), slow complementary sensitivity ('▷' yellow) and closed loop sensitivity ('◇' green) for the mid-ranging design.

$$\begin{aligned} U_s(z) &= q_{mr_s}(z)D(z) + q_{p_s}(z)U_r(z) \\ U_f(z) &= q_{mr_f}(z)D(z) + q_{p_f}(z)U_r(z). \end{aligned} \tag{22}$$

The mid-ranging design specifies not only the complementary sensitivity with both actuators, $T_{fs}(z)$, but also the complementary sensitivity corresponding to the control action with the slow actuator alone, $T_s(z)$ which are defined as follows,

$$\begin{aligned} T_{fs}(z) &= g_s(z)q_{mr_s}(z) + g_f(z)q_{mr_f}(z) \\ T_s(z) &= g_s(z)q_{mr_s}(z). \end{aligned} \tag{23}$$

To achieve the control objectives, $T_{fs}(z)$ and $T_s(z)$ must be unity at steady state and the decoupling between the setpoint on the faster input, U_r and Y is achieved through the use of pre-filters which must satisfy the condition,

$$g_s(z)q_{p_s}(z) + g_f(z)q_{p_f}(z) = 0. \tag{24}$$

Because $g_f(z)$ and $g_s(z)$ both include delay terms, $T_s(z)$ is chosen as

$$T_s(z) = T_s^-(z)T_s^+(z) \tag{25}$$

where $T_s^+(z)$ includes the delays of both $g_f(z)$ and $g_s(z)$ so that $T_s(z)/g_f(z)$ and $T_s(z)/g_s(z)$ are both causal and stable. So in this case,

$$T_s(z) = z^{-(d_s+d_f)} \frac{1 - \lambda_s}{1 - \lambda_s z^{-1}} \tag{26}$$

and the controller for the slow array is given by

$$q_{mr_s}(z) = T_s(z^{-1})/g_s(z)^{-1} \tag{27}$$

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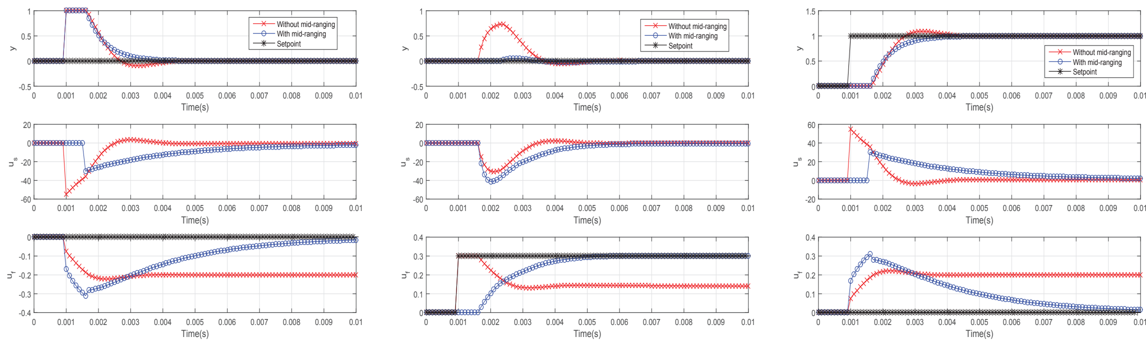

(a) Step in disturbance. (b) Step in fast actuator setpoint. (c) Step in output setpoint.

Figure 2: Responses of the output y and the slow u_s and fast u_f actuator inputs to various step changes when both actuators are used without mid-ranging control (‘x’ red) and with mid-ranging control (‘o’ blue). The reference for the output and fast actuator input are also shown (‘$*$’ black).

Likewise the fast and slow together sensitivity can be written as

$$T_{fs}(z) = T_{fs}^{-}(z)T_{fs}^{+}(z) \tag{28}$$

where $T_{fs}^{+}(z)$ includes the non-minimum phase components of $g_f(z)$ only so that $T_{fs}(z)/g_f(z)$ is causal and stable. The controller for the fast array is determined by

$$q_f(z) = \left[T_{fs}(z) - T_s(z)\right]/g_f(z) \tag{29}$$

Design of the pre-filters are described in [2] however a simple choice is

$$\begin{aligned} q_{p_f}(z) &= \tilde{q}_p(z)g_s^{+}(z) \\ q_{p_s}(z) &= -\tilde{q}_p(z)\frac{g_f(z)}{g_s^{-}(z)} \end{aligned} \tag{30}$$

where $\tilde{q}_p(z)g_s^{+}(z)|_{ss} = 1$ for some filter $\tilde{q}_p(z)$. The chosen complementary sensitivities are shown in Fig. 1 along with the closed loop sensitivities. The slow actuator has a control sensitivity that is low bandwidth only, while the fast actuator has a control sensitivity that is mid-frequency only and goes to zero at steady state; giving the mid-ranging effect. IMC is advantageous because it gives the control sensitivities of the fast and slow actuators directly as q_{mr_f} and q_{mr_s}.

In Fig. 2, the performance of the TISO system with and without mid-ranging is compared for a step change in disturbance, fast setpoint and output setpoint. Firstly, the effect of a step change in the disturbance is shown in Fig. 2a. The mid-ranging system firstly manipulates the fast actuator and moves the slow actuator to ensure that the fast actuator does not saturate by returning it to the setpoint $u_r = 0$. The non-mid-ranging system allows the fast input to settle to a steady state close to the saturation limit of ±0.5 which on the next upset, the fast actuator may easily because saturated. The responses to a step change in the fast actuator setpoint are shown in Fig. 2b and as before, the fast actuator achieves the requested setpoint change using the mid-ranging approach. Also, the effect in the output is decoupled through the pre-filters so that changes to the fast actuator setpoint are minimised on the output. Fig. 2c shows the responses to a change in the output setpoint. Both approaches achieve the required setpoint change, however the system without mid-ranging uses more control effort from the slower actuator. Mid-ranging control is therefore used to obtain a desired response from $Y(z)$ to $D(z)$ and from $U_r(z)$ to $U_f(z)$ and a decoupled response between $U_f(z)$ to $Y(z)$.

CONCLUSION

For a storage ring with two arrays of actuators used for electron orbit control, the overlap of the controllable subspaces of each array can be used to reduce the problem into either SISO, TISO or decoupled TITO blocks. The TISO blocks represent the modes of the system where both fast and slow actuators are required for control. For such cases, mid-ranging control is appropriate given the distinct characteristics of the two arrays of actuators and provides a control strategy that uses both actuators in such a way as to meet performance specifications without causing the faster actuator to saturate.

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